THE IMPORTANCE OF INCLUDING ELASTIC PROPERTY OF PENSTOCK IN THE EVALUATION OF STABILITY OF HYDROPOWEWR PLANTS

Torbjørn K. Nielsen

Dep. of Energy and Process Engineering, Water Power Laboratory, NTNU, Norway

ABSTRACT

To gain stable operation of a hydro power plant, it is mostly a matter of having the right ratio between the time constant of the rotating masses, Ta, and the time constant for the water masses, Tw. If Ta/Tw> 6 (or at least >4), the stability is normally not a problem. However, for power plants with long penstocks, this criterion is not enough. The elastic property of the penstock becomes an issue. The solution of the wave equation includes a term, which mathematically is defined as tanh (tangents hyperbolicus). This function is notorious unstable. It has a similarity to the tan-function, which goes from to ±∞ as it approaches ± 90 º. The cross frequency defines the frequency up to which the governor will function. Above the cross frequency, any disturbance will go without any interference from the governor. Therefore, the issue is to make sure that the elastic frequency is well above the cross frequency.

KEYWORDS

Water power, stability, penstock,elasticity

1. INTRODUCTION

Simulations with the purpose of controlling system stability is preferably done in the frequency domain, because then all the eigen frequencies of the system are identified. If there is a stability problem, the cause can easily be detected. This is not the case if one do the simulations in time domain.

The differential equations for the system must be linearized around the point of operation, i.e. at a given flow Q_0, head, H_o and speed of rotation n_0, and then Laplace transformed to frequency domain. The system is hereby defined by it's transfer functions presented in a block diagram. This is a well documented method. There are numerous methods to analyse the result by graphical representation, like Bode, Nyquist, Nichols, Root-locus to mention a few. They all ends up finding the stability margins, i.e. the Phase margin and the Gain margin.

The transfer function of a hydro power plant assumed rigid penstock is shown in Fig. 1. The most important time constants are T_w and T_a. T_w is the time constant for the inertia of the water masses, which is defined as the time it takes to accelerate the water masses from zero to nominal flow. The water masses participating in the governing process is from the nearest surface up-stream the turbine to the nearest water surface down-stream the turbine. T_a is the time constant for the rotating masses, which is defined as the time it takes to accelerate the rotating masses, i.e. mainly the generator, from zero to nominal speed of rotation with full torque.

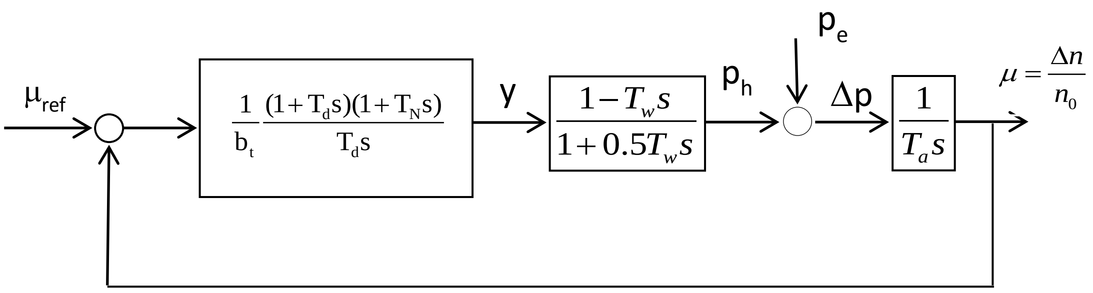

Figure 1: Block diagram for hydro power plant

T_w can be derived using Newton's 2. Law:

$$T_w = \frac{Q_0}{H_0}\frac{L}{gA} \tag{1}$$

T_w is highly dependent on the hydraulic design of the power plant, while T_a is more or less given by the generator manufacturer.

The transfer function between the guide vane position, y, and the hydraulic power, p_h, is in more detail shown in Figure 2.

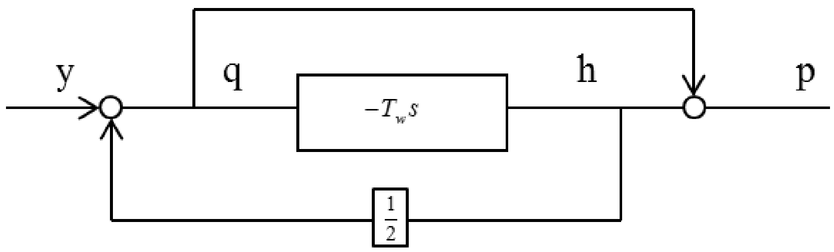

Figure 2 Block diagram between guide vane position y and power, p, rigid penstock

If one include the elastic property in the penstock, the solution of the Allievi equation [3] will include a tanh-function; hence, the block diagram will be as shown in Fig. 3. The constant, h_w, is the Allievi constant, which is defined as:

$$h_w = \frac{Q_o a}{2AgH_o} \tag{2}$$

The normal criterion is that if $h_w>1$, preferable a good deal bigger than 1, the elasticity can be disregarded.

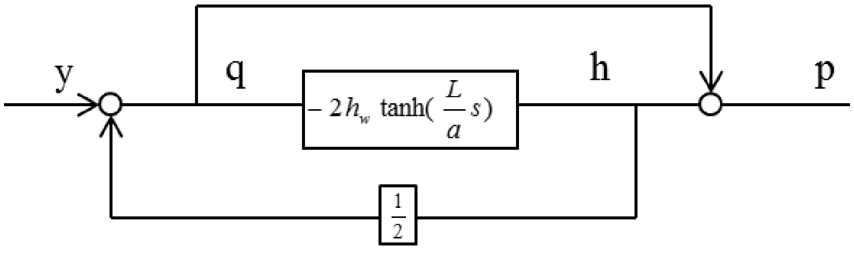

Figure 3 Block diagram between guide vane position, y, and hydraulic power, p_h. Elastic penstock.

The Allievi constant is in fact the ratio T_w/T_r, where T_r is the reflection time of the elastic wave:

$$T_r = \frac{2L}{a} \tag{3}$$

Where L is the length of the penstock an a is the pressure propagation speed or the speed of sound. The criterion $h_w>1$ can then be interpreted as:

$$\frac{T_w}{T_r} > 1 \text{ or } \frac{1}{T_r} > \frac{1}{T_w} \tag{4}$$

The cross frequency is very near $1/T_w$, so $h_w>1$ means that the frequency for the elastic waves must be higher, or even far higher than the cross frequency. In that case, the governor will not react on the elastic waves, and the elasticity will not be a stability issue.

2. COMPARISON OF THE RIGID AND ELASTIC TRANSFERE FUNCTION

The two block diagrams shown in Fig. 2 and Fig. 3, gives the two following transfer functions between guide vane position and power:
Rigid:

$$\frac{p}{y} = \frac{1 - T_w s}{1 + 0.5 T_w s} \tag{5}$$

Elastic:

$$\frac{p}{y} = \frac{1 - 2h_w \tanh(\frac{L}{a} s)}{1 + h_w \tanh(\frac{L}{a} s)} \tag{6}$$

Where s is the complex variable: $s = j\omega$.

According to methods described in control theory [1, 2] the amplitude and the phase angle can be solved and gives an illustration of the difference between rigid and elastic representation. In general, for a transfer function:

$$A(s) = A(j\omega) = \frac{(1+T_1 s)(1+T_2 s)}{(1+T_3 s)(1+T_4 s)} \tag{7}$$

The amplitude is:

$$|A(j\omega)| = \sqrt{\frac{(1+(\omega T_1)^2)(1+(\omega T_2)^2)}{(1+(\omega T_3)^2)(1+(\omega T_4)^2)}} \tag{8}$$

The phase angle is:

$$\angle A(j\omega) = \operatorname{atan}(\omega T_1) + \operatorname{atan}(\omega T_2) - \operatorname{atan}(\omega T_3) - \operatorname{atan}(\omega T_4) \tag{9}$$

For the rigid transfer function, the calculation is rather straight forward. For the elastic transfer function, the complex variable s in the expression $\tanh(\frac{L}{a}s)$ makes a problem. The complex "j" has to be outside the parenthesis. Using Euler equation $e^{j\phi} = \cos\phi + j\sin\phi$, illustrated in Fig. 4, makes it possible to rearrange the equation.

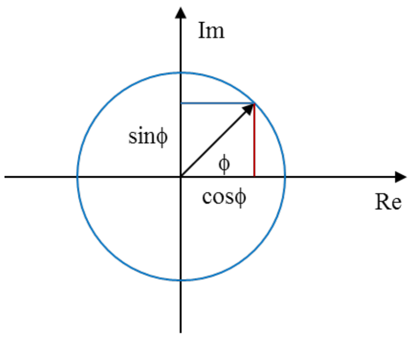

Figure 4 Illustration of the *Euler equation*

The Euler equation for this particular case:

$$e^{j\frac{L}{a}\omega} = \cos(\frac{L}{a}\omega) + j\sin(\frac{L}{a}\omega) \tag{10}$$

Inserted in the equation for the tanh term, mathematically defined as:

$$\tanh(\frac{L}{a}s) = \frac{(e^{\frac{L}{a}s} - e^{-\frac{L}{a}s})}{(e^{\frac{L}{a}s} + e^{-\frac{L}{a}s})} \qquad \text{where } s=j\omega \tag{11}$$

$$\tanh(\frac{L}{a}j\omega) = \frac{\cos(\frac{L}{a}\omega) + j\sin(\frac{L}{a}\omega) - \cos(-\frac{L}{a}\omega) - j\sin(-\frac{L}{a}\omega)}{\cos(\frac{L}{a}\omega) + j\sin(\frac{L}{a}\omega) + \cos(-\frac{L}{a}\omega) + j\sin(-\frac{L}{a}\omega)} \tag{12}$$

$$\tanh(\frac{L}{a}s) = j\tan(\frac{L}{a}\omega) \tag{13}$$

The tanh(L/as) term has transformed to j tan(L/aω) and the amplitude and angle can be calculated.

For the rigid transfer function:

$$\text{Amplitude:} \qquad |A(j\omega)| = \sqrt{\frac{(1+(\omega T_w)^2)}{(1+(0.5\omega T_w)^2}} \tag{14}$$

Phase angle: $$\angle A(j\omega) = -\mathrm{atan}(\omega T_w) - \mathrm{atan}(0.5\omega T_w) \tag{15}$$

For the elastic transfer function:

Amplitude: $$\left|A(j\omega)\right| = \sqrt{\frac{(1+(2h_w \tan(\frac{L}{A}\omega))^2}{(1+(h_w \tan(\frac{L}{A}\omega))^2}} \tag{16}$$

Phase angle: $$\angle A(j\omega) = -\mathrm{atan}(2\mathrm{h}_w \tan(\frac{L}{A}\omega)) - \mathrm{atan}(\mathrm{h}_w \tan(\frac{L}{A}\omega)) \tag{17}$$

Solving the equations for increasing ω, the result is shown in Figure 5 and Figure 6, left Figure rigid and right Figure elastic property.

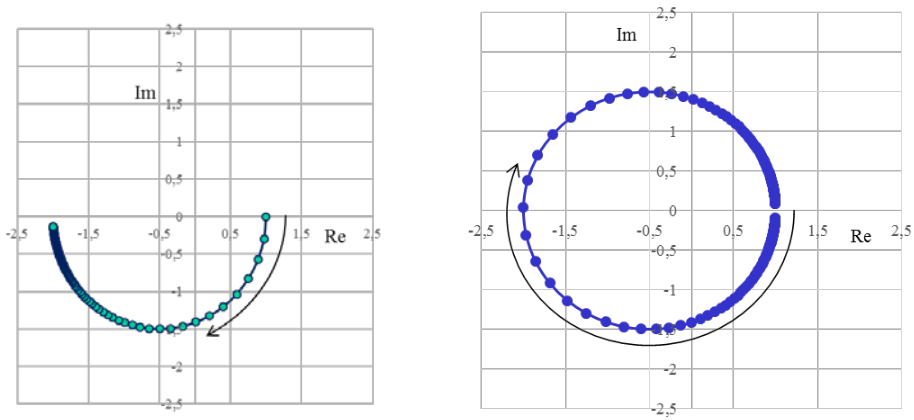

Figure 5 Rigid (left) and elastic transfer function plotted in a Re-Im plane.

The difference is that the rigid function goes from 1 and stops at -2 on the Re-axes, while the elastic function takes the whole turn all the way back to 1 again. The intuitive explanation of this is that the rigid just stops, while the elastic one bounces back due to the elasticity.

Figure 6 shows the same performance in a Bode plot (Amplitude and Phase vs frequency)

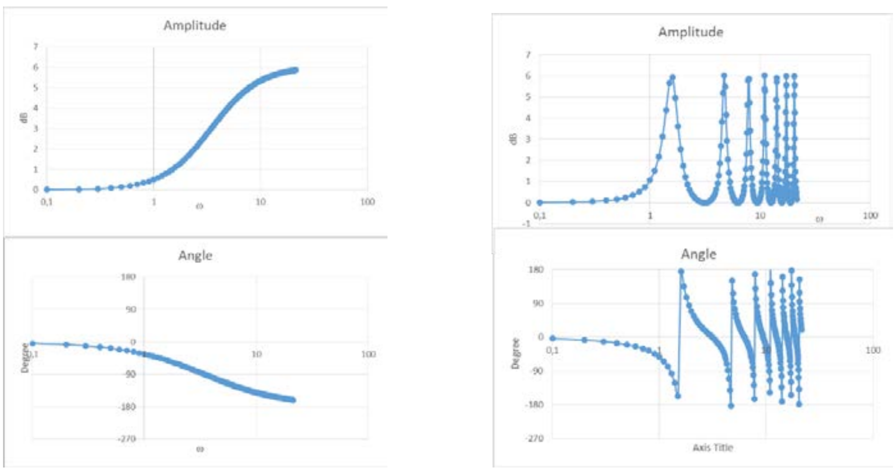

Figure 6 Bode plot for rigid and elastic penstock

3. CONSEQUENCE FOR THE STABILITY SIMULATION OF A POWER PLANT

To establish if a power plant is stable or not, is a question of checking the stability margins, i.e. the amplifying margin and the Phase margin. The complete transfer function of the power plant must be established and the amplitude and phase angle calculated as a function of the frequency. There are many ways of plotting the result in order to find a conclusion on whether the stability margin is satisfactory. The author prefers the Bode plot, however to plot the result in a Re-Im plan, Nyquist diagram, might give additional information.

The complete transfer functions for rigid and elastic model is:

Rigid: $$A(j\omega)=\frac{1}{b_t T_d T_a}\frac{(1+T_d s)(1+T_N s)}{s^2}\frac{(1-T_w s)}{(1+0.5T_w s)} \quad (18)$$

Elastic: $$A(j\omega)=\frac{1}{b_t T_d T_a}\frac{(1+T_d s)(1+T_N s)}{s^2}\frac{(1-2h_w\tanh(\frac{L}{a}s))}{(1+h_w\tanh(\frac{L}{a}s))} \quad (19)$$

For the rigid function the Amplitude is:

$$\left|A(j\omega)\right|=\frac{1}{b_t T_d T_a}\sqrt{\frac{1+(\omega T_d)^2)(1+(\omega T_N)^2}{\omega^4}\frac{1+(\omega T_w)^2)}{1+(0.5\omega T_w)^2)}} \quad (20)$$

And the Phase angle is:

$$\angle A(j\omega)=\mathrm{atan}(\omega T_d)+\mathrm{atan}(\omega T_N)-\mathrm{atan}(\omega T_w)-\mathrm{atan}(0.5\omega T_w)-\pi \quad (21)$$

Because of the negative sign in the expression $(1-T_w s)$ in the numerator of eq.18, the phase shift is negative. This is the key issue regarding stability and T_w. It makes the phase go towards -180^0. The $-\pi$ term in eq. 21 comes in because of the two poles at s=0.
For the elastic function the Amplitude is:

$$\left|A(j\omega)\right|=\frac{1}{b_t T_d T_a}\sqrt{\frac{1+(\omega T_d)^2)(1+(\omega T_N)^2}{\omega^4}\frac{1+(2h_w\tan(\frac{L}{a}\omega))^2}{1+(h_w\tan(\frac{L}{a}\omega))^2}} \quad (22)$$

And the phase angle is:

$$\angle A(j\omega)=\mathrm{atan}(\omega T_d)+\mathrm{atan}(\omega T_N)-\mathrm{atan}(2h_w\tan(\frac{L}{a}\omega))-\mathrm{atan}(h_w\tan(\frac{L}{a}\omega))-\pi \quad (23)$$

Figure 8 shows Bode plots for both rigid an elastic model. Ignoring the elasticity of the penstock, the hydro power plant seems to be stable as both Phase margin and Gain margin is sufficient. Including the elasticity, the system is on the border of instability, as shown in Fig. 8, right.

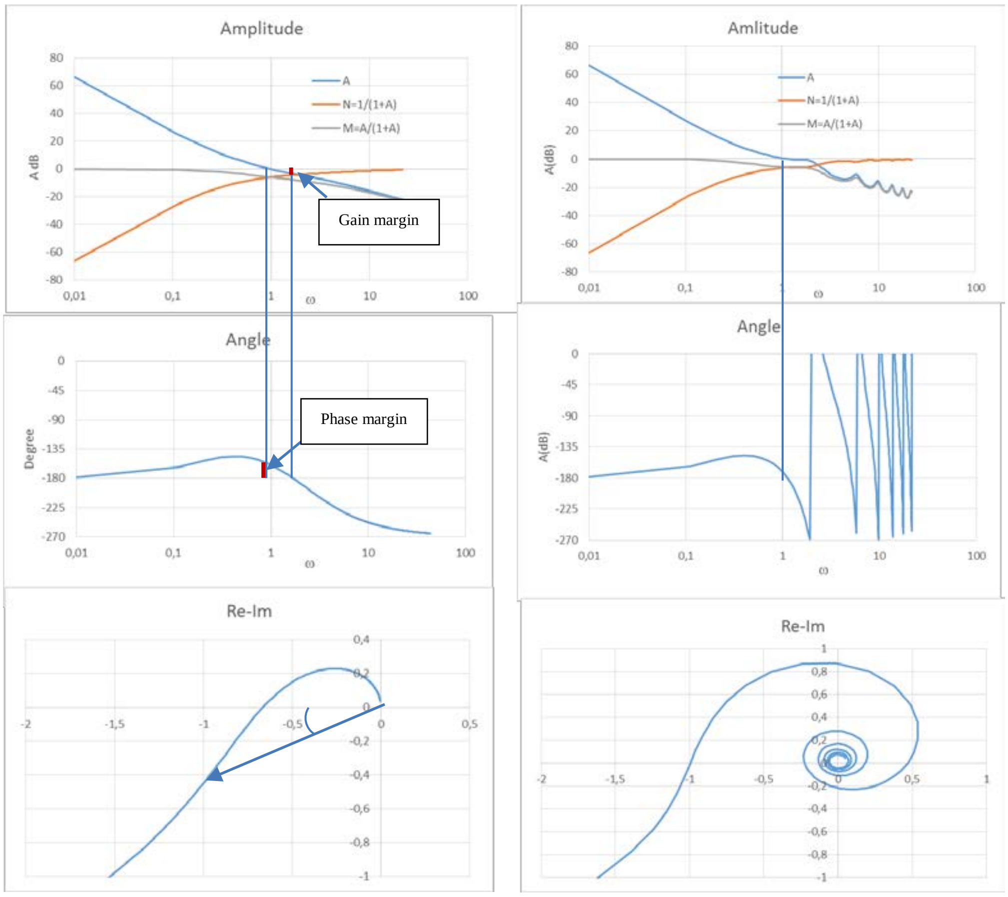

Figure 8: Bode plots. Left side shows the rigid simulation and right side the elastic simulation, T_w = 0,71sec, T_a=6sec, h_w=0.45. In the upper Figures, A is open loop, M is closed loop and N is the sensitivity. The two figures at the bottom shows the same, plotted in Re-Im plane (Nyquist diagram)

The PID parameters used for the simulations shown in Figure 8 are b_t=0.1, T_d=6sec, T_N=0.0. It is of course possible to stabilize the system by tuning the PID parameters, however, the cross frequency will easily be at too low frequency, which means that the governing will be to slow and standing oscillations will occur. Increasing the inertia of the generator, i.e. increasing T_a will have the same effect.

4. CONCLUSIONS

High head power plants have often long penstocks, which makes the reflection time, T_r, too big compared to T_w. It is quite possible to achieve a T_w<1, which is often the design criterion used, and still get instability because of the elastic property of the penstock. In order to design a stable system with sufficient stability margins, Allievi's constant must be checked by calculating the ratio T_w/T_r . This ratio, Allievi's constant, should be at least bigger than 1.

$$\frac{T_w}{T_r} > 1 \quad \text{i.e.} \quad \frac{1}{T_r} > \frac{1}{T_w} \tag{24}$$

Which means that the frequency of the elastic wave must be bigger than the cross frequency, which is very near $1/T_w$.

If this criterion is not fulfilled, it might still be possible to make the system stable by optimizing the governor settings; however, the quality of the governing system will be lousy.

5. REFERENCES

[1] Norman S. Nise: *Control Systems Engineering*,The Benjamin/Cummings Publishing Company, Inc., ISBN 0-8053-5420-4

[2] Balchen, Fjeld, Solheim: *Reguleringsteknikk*, TAPIR, ISBN 82-519-0626-1

[3] Brekke, H: A Stability Study on Hydro Power Plant Governing Including the Influence from a Quasi Nonlinear Damping of Oscillatory Flow and from the Turbine Characteristics, PhD dissertation NTNU, 1984

6. NOMENCLATURE

T_w	(s)	Penstock time constant	a	($m.s^{-1}$)	Pressure propagation speed
T_a	(s)	Time constant for rot. masses	L	(m)	Length of penstock
T_d	(s)	Dash-pot time constant	h_w	(-)	Allievie's constant
T_N	(s)	Derivative time constant	s	(-)	Complex variable
T_r	(s)	Reflection time	q	(-)	Dimensionless flow
b_t	(-)	Transient speed droop	h	(-)	Dimensionless head
y	(-)	Guide vane position	p	(-)	Dimensionless power
μ	(-)	Dimensionless speed of rotation			