6th IAHR International Meeting of the Workgroup on Cavitation and Dynamic Problems in Hydraulic Machinery and Systems, September 9-11, 2015, Ljubljana, Slovenia

MODEL FOR CAVITY DESCRIPTION IN A FRANCIS TURBINE DRAFT TUBE

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ABSTRACT

Among the peculiarities of the hydro power stations one can highlight oscillations of the water reservoir level depending on the seasons and amount of rains from one side and non-uniform electrical load required by the recipients from the other side. Thus, the hydraulic machines should operate under wide range of parameters, that is one of the reason leading to the cavitation developments inducing undesirable unsteady phenomena on the hydropower plant. The hydrodynamic and hydroacoustic instabilities of various types are strongly influenced by the geometrical parameters of the cavities arising behind the hydroturbine runner. For the instabilities analysis one needs also in the cavitation compliance and the flow gain factor. The cavitation compliance represents the cavitation volume variation with the respect to a variation of pressure and defines implicitly the local wave speed in the draft tube. And the flow gain factor reflects the rate of the cavitation volume changing with the flow rate variation. Thus, it would be attractive to develop some analytical or semi-empirical models describing the form and volume of cavities in dependence on the pressure and flow rate for various operation points.

In the present study, the cavity model constructing is based on the conservation laws for fluxes of mass, momentum and moment of momentum in the draft tube cone. In real flows the conservation laws are not fulfilled completely. Here, first, we checked out fulfillment of the conservation laws with use of the previous experimental data on the velocity field. We found the real variations of the fluxes along the cone in the cases of cavitation free conditions. These dependencies are taken as a base to solve the problem on the cavitation area geometry for the specified operation points.

KEYWORDS

Cavitation, vortex rope, swirl, conservation laws

1. INTRODUCTION

The hydro power stations serve not only as the source of electricity. They are found to be most proper regulating instrument in electric grids. In the same time the water levels in the upper and lower biefs undergo considerable variations due to seasons changings or meteorological conditions. These factors are the reasons why the hydraulic machines need to operate under wide range of technological parameters. Most of the operating points lying

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away from the best efficiency point are subjected to instability or cavitation developments inducing undesirable unsteady phenomena on the hydropower plant. In a most extent this refers to the turbines of Francis type with the runner having hardly mounted blades. The cavitation phenomenon arises when the local pressure decreases lower then the pressure of saturated water vapor. The hydrodynamic and hydroacoustic instabilities of various types are strongly influenced by the geometrical parameters of the cavities arising behind the hydroturbine runner. The review on researches of instabilities coused by the cavitation can be found in recent works [1], [2]. Usually researches apply the experimental [3] or CFD [4] modeling of the vortex rope with cavitation. For the instabilities analysis one needs in the cavitation compliance and flow gain factor. The cavitation compliance represents the cavitation volume variation with the respect to a variation of pressure and defines implicitly the local wave speed in the draft tube. And the flow gain factor reflects the rate of the cavitation volume changing with the flow rate variation.

The experimental or numerical approaches for determination of the cavitation compliance and flow gain factor require high enough resources and do not obey necessary accuracy. Thus, it would be attractive to develop some analytical or semi-empirical models describing the form and volume of cavities in dependence on the operation point. In the present study, the cavity model constructing is based on the conservation laws for fluxes of mass, vorticity, momentum and moment of momentum in the draft tube cone. The approach with use of the conservation laws was sucessfully used during development of the model of precessing helical vortex in conical part of the hydroturbine draft tube [5]. In the real flows the conservation laws are not fulfilled completely. Here, first, we checked out their fulfillment with use of previous data on the velocity field obtained on the reduced scale model of a Francis turbine [2], [6], [7]. The velocity measurements were done in few cone cross-sections with LDA and/or PIV systems. We found the real variations of the fluxes along the cone in the cases of cavitation free conditions and in some special operation points with steady axisymmetric cavity. These dependencies allowed us to solve the problem on the cavitation area geometry for the specified operation points.

2. CONSERVATION LAWS IN THE DRAFT TUBE FLOW

The experimental data for the analysis of the fluxes conservation were obtained by Müller [2] at the test setup representing a 1:16 reduced scale physical model of a Francis turbine with a specific speed of $\nu = 0.27$ installed on the EPFL test rig PF3 of the Laboratory for Hydraulic Machines (Fig.1).

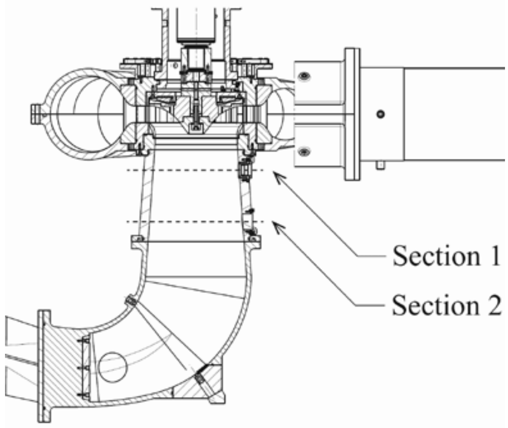

Fig.1. View of the reduced scale model. Section 1 and Section 2 for LDV measurements are located $0.39 \times D_{\bar{1}}$ and $1.0 \times D_{\bar{1}}$ below the runner exit

The axial (meridional) and tangential velocity components C_m and C_u were measured using Laser Doppler Velocimeter (LDV) of Dantec FlowExplorer type in two cross-sections in the draft tube cone Section 1 and Section 2 as shown in Fig.1. Here we consider operation points corresponding to the axisymmetrical flow in the draft tube cone: the Best Efficiency Point (BEP) with parameters n_{ED} = 0.288, Q_{ED} = 0.200, σ = 0.11, N = 800 min^{-1} and the overload operation point with parameters n_{ED} = 0.288, Q_{ED} = 0.253, σ = 0.11, N = 650 min^{-1}.

Figure 2 shows the mean axial velocity profiles measured by LDV in two cross-sections at the BEP in cavitation free conditions [2]. The velocity is made non-dimensional with the mean discharge speed in the given measurement section. The vertical bars in plots correspond to the standard deviation values. As seen the highest measurement errors lie in the near-wall zone due to problems of optical access. From comparison of the mirror-reflected profile with itself (see Fig.3) one can conclude that the profiles are not centered. For example, profile of the axial velocity presented in Fig.2a should be shifted to left by 0.011. To perform the further analysis it will be proper to symmetrize and smooth profiles. Moreover, due to the turbulent character of the flow (Reynolds number is of order 10^6) we can approximate the velocities in the near-wall zone by the function

$$k(R-|x|)^{-1/7}.. \qquad (1)$$

The coefficient k has being chosen to fit the experimental data. The results of the data treatment are presented in Fig.4 by solid lines, now, as radial profiles of the axial and tangential velocity components for Section 1 and Section 2.

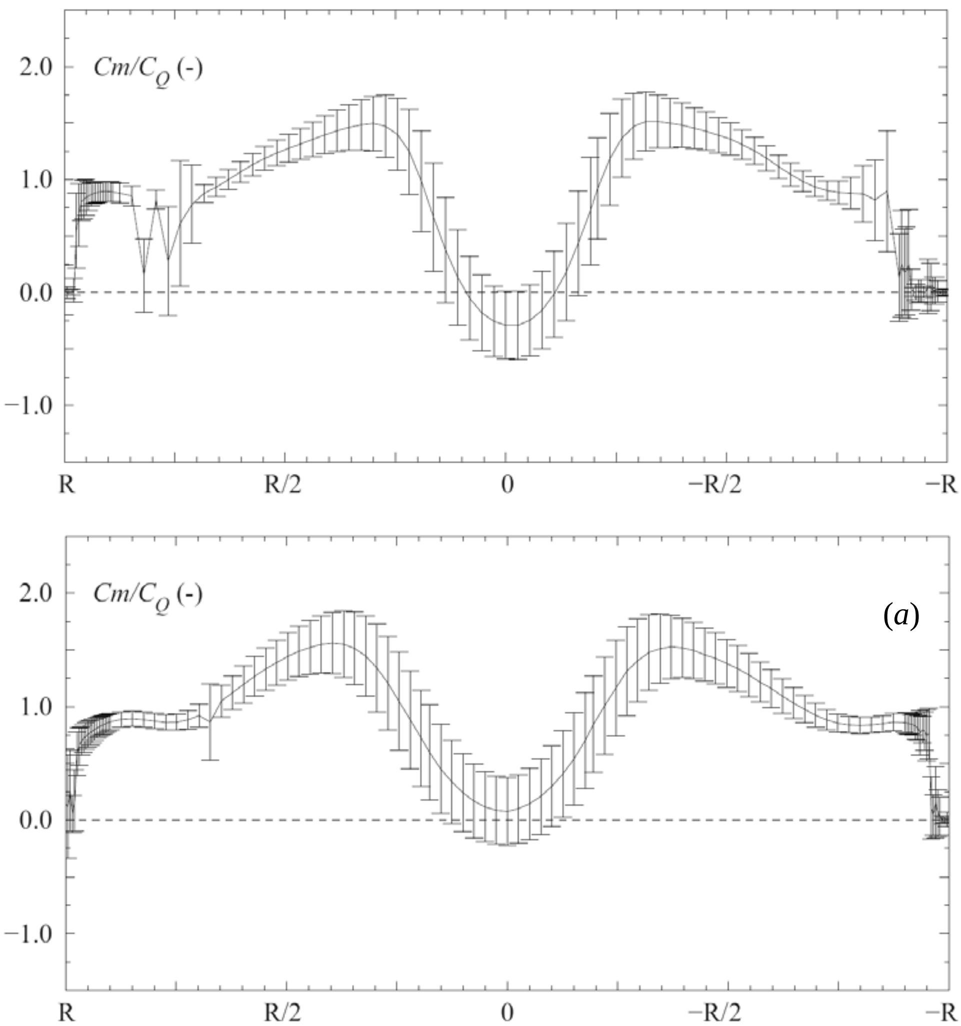

Fig.2. Profiles of the axial velocity component C_m at the BEP [2] at Section 1 (a) and Section 2 (b)

(*b*)

The above actions are undertaken to prepare the checking for validness of the laws of conservation of the hydrodynamic fluxes. Among the quantities which should be the same from one cross-section to another we consider, first of all, the flow rate

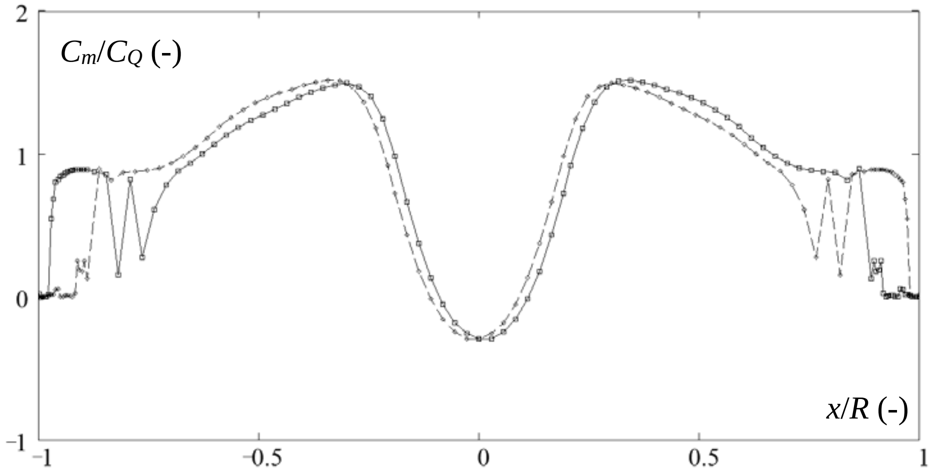

Fig.3. Checking for symmetry of the axial velocity distribution from Fig.2a

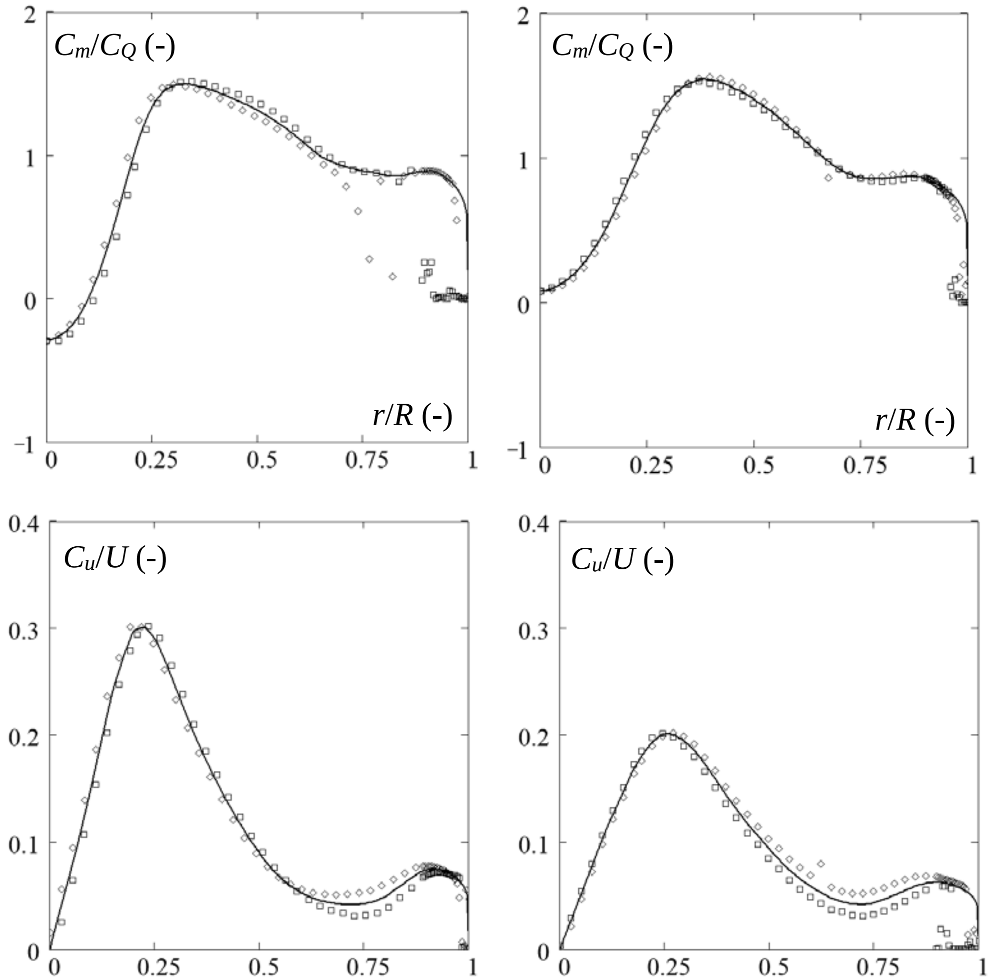

Fig.4. Symmetrized and smoothed profiles of the axial (a, b) and tangential (b, c) velocities in Section 1 (a, c) and Section 2 (b, d) of the draft tube. Symbols correspond to the measurements data from [2].

$$Q = 2\pi \int_0^{R(z)} rC_m(r,z)dr = const. \tag{2}$$

Next quantity, the flux of momentum,

$$F_m = 2\pi \int_0^{R(z)} rC_m^2(r,z)dr. \tag{3}$$

can be conserved in laminar channel flows. The third quantity is the flux of moment of momentum in full statement should also contain the impact of turbulent pulsations:

$$F_{mm} = 2\pi \int_0^{R(z)} r^2 C_m C_u dr = const. \tag{4}$$

More rigorous formulation of the conservation law for flux of momentum takes into account distribution of the pressure and turbulent pulsations both for F_m and F_{mm} [8]. These three laws can be accompanied by the condition of energy balance, the Bernoulli equation

$$\frac{p}{\rho} + gz + \frac{1}{2}C_m^2 = const. \tag{5}$$

In a swirl flow, like in hydroturbine draft tube, the pressure distribution is strongly non-uniform and pressure should be replaced by the averaged quantity

$$p_{av}(z) = \frac{2}{R(z)^2} \int_0^{R(z)} rp(r,z)dr. \tag{6}$$

The pressure profile can be found by integration

$$p(r,z) = p_0(z) + \int_0^r \frac{1}{r}C_u^2(r',z)dr', \tag{7}$$

where $p_0(z)$ is the pressure at the cone axis. The velocity in Eq.(6) also has the averaged character, it is discharge mean speed. Thus, the Bernoulli equation can be written finally as

$$\frac{p_{av}(z)}{\rho} + gz + \frac{1}{2}\left(\frac{Q}{\pi R(z)^2}\right)^2 = const. \tag{8}$$

One more quantity that should conserve its value in a channel swirl flow is the flux of vorticity, but in the case of axi-symmetrical flow this flux is identically equals zero.

Calculation of the flow rates in Section 1 and Section 2 with the data presented in Fig.4 gave good result with the relative difference $(Q_2 - Q_1)/Q_2 = 0.0049$. During evaluation of the fluxes of momentum by Eq.(2) the difference is more essential, 14 %. The reason, obviously, lies in neglecting the turbulence impact. For the fluxes of moment of momentum the result is much better, 0.9 %. For the second operation point corresponding to the overload condition we have not found fulfillment of the flow rate conservation. This fact requires a special analysis and further we will limit our consideration by the Best Efficiency Point.

3. MODEL OF CAVITATIONAL BUBBLE IN THE DRAFT TUBE CONE

When the pressure behind the hydroturbine runner becomes less than the water vapour pressure p^{wvp}, a cavitational bubble arises in the draft tube. To construct a model of cavitational bubble we take known velocity field (say, at BEP) and consider a situation when the pressure in the draft tube decreases. In a real hydro power station this corresponds to growth of the suction height. At the current stage we know the velocity field and integral parameters at two cross-sections. The flow rate Q and the flux of moment of momentum are practically the same in these sections. Suppose that the flux of momentum (without turbulent member) changes linearly inside the conical part of draft tube. When the cavitational bubble arises the fluxes laws remain unchanged due to three order difference in density of water and vapor. Thus, together with the Bernoulli equation we will have four equations for the cavity bubble description. Next, to derive these equations consider parameterization of the velocity profiles. Any profile presented in Fig.4 obeys two maxima and one minimum. Such form allows us to apply piecewise parabolic functions matched with functions of Eq.(1) type in near-wall zone:

$$C(r)=\begin{cases} a_0 r^2 + b_0 r + c_0, \ 0 \le r < r_{01} \\ a_1 (r-r_1)^2 + b_1, \ r_{01} \le r < r_{12} \\ a_2 (r-r_2)^2 + b_2, \ r_{12} \le r < r_{23} \\ a_3 (r-r_3)^2 + b_3, \ r_{23} \le r < r_{34} \\ a_4 (R-r)^{1/7}, \ r_{34} \le r < R \end{cases} \quad (10)$$

Here r_i, i = 1, 2, 3 are the coordinates of extrema, r_{ij} are the points of matching of individual pieses. In these points we require matching of the derivatives too. For the axial velocity profiles coefficient $b_0 = 0$, for the tangential velocity $a_0 = 0$, $c_0 = 0$. Thus, for parametrization of an axial velocity profile one needs in eight parameters: $r_1^m, r_2^m, r_3^m, r_{01}^m, r_{34}^m, a_0^m, c_0^m, a_4^m$. All other parameters have being determined trough these eight due to matching conditions. For a tangential velocity it is enough seven parameters: $r_1^u, r_2^u, r_3^u, r_{01}^u, r_{34}^u, b_0^u, a_4^u$. The values of parameters corresponding to the profiles shown in Fig.4 are presented in Tab.1.

	a_0	b_0	c_0	r_1	r_2	r_3	r_{01}	r_{34}	a_4
C_m, Section 1	$9.1\ 10^{-4}$	–	–0.29	0.321	0.888	0.926	0.192	0.931	0.875
C_m, Section 2	$4.7\ 10^{-4}$	–	0.085	0.403	0.840	0.888	0.195	0.908	0.848
C_u, Section 1	–	0.0080	–	0.225	0.717	0.918	0.192	0.948	0.0717
C_u, Section 2	–	0.0049	–	0.262	0.741	0.908	0.151	0.918	0.0623

Tab.1 The values of parameters of the piecewise parabolic function approximating the velocities profiles in Section 1 and Section 2 at BEP

Since we are trying to derive system of four equation, it is reasonable to have four unknowns. At every cross-section of the cone we suppose that velocities profiles have forms which can be parameterized by the function in Eq.(10). Here we suggest that all the coordinate points r_i and r_{ij} are linear functions of z, i.e. from the Tab.1 we will find positions these points for any cross-section. The same linear dependence is proposed for the coefficient a_4 responsible for the near-wall function. As for coefficients a_0, b_0, c_0 and the cavity radius r_c, they will serve as the unknowns.

Finally, in view of Eqs.(2, 3, 5, 8) we write the system of equations for the cavity bubble description

$$G_Q(z, r_c, a_0, c_0) = \frac{2}{R(z)^2} \int_{r_c}^{R(z)} r C_m^p \left(r, a_0, c_0, a_4^{mz}, r_{01}^{mz}, r_{34}^{mz}, r_1^{mz}, r_2^{mz}, r_3^{mz}\right) dr - 1 \quad (11)$$

$$G_M\left(z,r_c,a_0,c_0\right)=\frac{2}{\pi R(z)^4}\int_{r_c}^{R(z)} r\left[C_m^p\left(r,a_0,c_0,a_4^{mz},r_{01}^{mz},r_{34}^{mz},r_1^{mz},r_2^{mz},r_3^{mz}\right)\right]^2 dr-F_m(z) \quad (12)$$

$$G_{MM}\left(z,r_c,a_0,b_0,c_0\right)=\frac{2}{R(z)^2}\int_{r_c}^{R(z)}\left[r^2C_m^p\left(r,a_0,c_0,a_4^{mz},r_{01}^{mz},r_{34}^{mz},r_1^{mz},r_2^{mz},r_3^{mz}\right)\times C_u^p\left(r,b_0,a_4^{uz},r_{01}^{uz},r_{34}^{uz},r_1^{uz},r_2^{uz},r_3^{uz}\right)\right]dr-F_{mm} \quad (13)$$

$$G_P\left(z,r_c,b_0\right)=F_P\left(z,r_c,b_0\right)-F_P\left(z_1,r_{c1},b_{01}\right), \quad (14)$$

$$F_P\left(z,r_c,b_0\right)=\frac{1}{\rho}p_{av}^p\left(z,r_c,b_0\right)+\frac{gD_1}{U^2}z+\frac{Q^2}{2\pi^2U^2R(z)^4}$$

Here the superscripts with z denote that these parameters are known functions of axial coordinate z. The superscript 'p' means the use of piecewise parabolic function.

The derived system of equations Eqs.(11 – 13) consists of complex non-linear equations. The most proper way to solve it lies in composing a quadratic form

$$G_Q\left(z,r_c,a_0,c_0\right)^2+\left[\frac{G_M\left(z,r_c,a_0,c_0\right)}{F_m(z_1)}\right]^2+\left[\frac{G_{MM}\left(z,r_c,a_0,b_0,c_0\right)}{F_{mm}}\right]^2+\left[\frac{G_P\left(z,r_c,b_0\right)}{F_p\left(z_1,r_{c1},b_{01}\right)}\right]^2 \quad (15)$$

and minimizing it by the Gradient Descent Method. The value of cavity radius r_{c1} is taken as an initial value. And first we find coefficients a_0, b_0, c_0 at Section 1. Then we use found quantities as an initial approach in the closest cross-sections $z_1 + \Delta z$ and $z_1 + \Delta z$, and so on.

The examples of calculated forms of the cavitational bubble in the draft tube are shown in Fig.5. They look very similar to the forms observed in experiments or calculated with CFD [2], [5], [6].

4. CONCLUSION

The next step is done on the way for development of theoretical and semi-empirical approaches for modelling the cavitational structure arising in swirl flow behind the hydroturbine runner. The present approach is based on the idea for using the conservation laws for the model constructing. The analysis of experimental data confirmed fulfilment of conservation of the flux of mass (flow rate) and flux of moment of momentum. As for flux of momentum, we used linear approximation for changing of this quantity through the draft tube cone. Further parameterization of the velocity field allowed for derivation of the system of equations describing the form of cavitational bubble. The forms obtained during solution of the problem with the method of gradient descent look very similar to cavities observed in experiments.

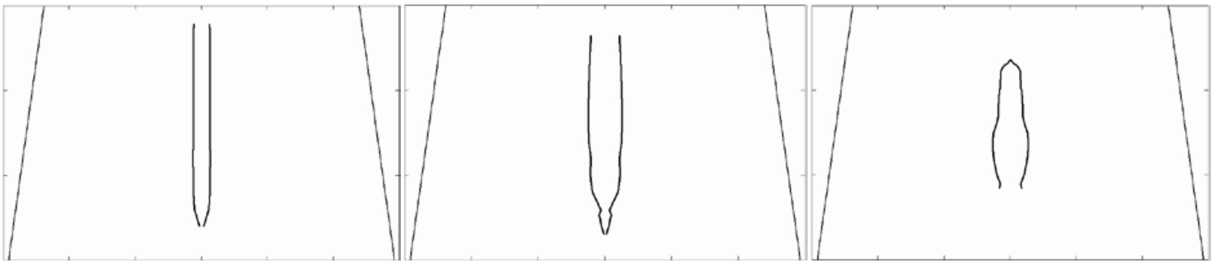

Fig.5. Forms of calculated cavitational bubbles in the draft tube

5. ACKNOWLEDGEMENTS

Prof. P. Kuibin (in part related to the model development and calculations) was supported by RSF (project No. 14-29-00093).

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7. NOMENCLATURE

C_m	(m.s^{-1})	axial (meridional) velocity component	Q	(m^3.s^{-1})	flow rate
C_u	(m.s^{-1})	tangential velocity component	R	(m)	radius
D_1	(m)	diameter of runner outlet	r	(m)	radial coordinate
F_m	(m^4.s^{-2})	flux of momentum	x	(m)	coordinate 'UB – LB'
F_{mm}	(m^5.s^{-2})	flux of moment of momentum	y	(m)	coordinate 'RBr – LBr'
g	(m.s^{-2})	acceleration due to gravity	z	(m)	axial coordinate
p	(Pa)	pressure	ρ	(kg.m^{-3})	density