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ANALYSIS OF NATURAL FREQUENCIES OF DISC-LIKE STRUCTURES IN WATER ENVIRONMENT BY COUPLED FLUID-STRUCTURE-INTERACTION SIMULATION

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ABSTRACT

The dynamic behavior of disc like structures in water environment by means of two-way fluid-structure-coupled simulation is presented. It is well known that the natural frequencies of structures change by transfer from air to water environment due to the added mass effect. Recent experimental investigation [1] has confirmed an analytical study [2] that an additional shift in natural frequencies is caused by rotary motion of the disc. This shift in natural frequency becomes important for resonance check of hydraulic turbine runners. For pump turbines or high head Francis turbines e.g. crown and band are similar to discs. Those runners are characterized by crown or band dominated mode shapes with natural frequencies that might be close to excitation frequencies of rotor stator interaction. For this purpose, accurate knowledge of natural frequencies is essential.
In this talk results of two-way fluid-structure-coupled simulation of a rotating disc in contact with water using the commercial software ANSYS® mechanical combined with ANSYS® CFX are given for different rotating speeds. The shift in natural frequencies is compared to analytical as well as experimental results.

KEYWORDS

Fluid-structure-interaction, coupled simulation, natural frequencies, mode shapes

1. INTRODUCTION

Rotating components in turbomachinery are subjected to excitation from the interaction of rotating parts with stationary components. For reliable operation of rotating machines the natural frequencies should be sufficiently separated from excitation frequencies. Due to the properties of water this criterion is crucial in hydraulic turbomachinery. Therefore, accurate knowledge of the natural frequencies is essential. Especially the natural frequencies of diametrical modes are of interest because of the shape of the excitation source.
For the calculation of natural frequencies of hydraulic components the surrounding water as well as the casing has to be considered. At first, the water causes a decrease of the natural frequencies because of the added mass effect. Here, the geometry of the casing has an impact on this effect. Second, the natural frequencies are influenced by fluid flow induced by rotation of the turbine runner. This effect is under special investigation within this proceeding.

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For a more general investigation of the basic principle of the effect, a simplified model of a hydraulic turbine is considered – namely a disc. This simplification is adequate since crown and band of high head Francis turbine runners and pump turbine runners are similar to discs, cf. Fig. 1.

Fig. 1: Picture of a pump turbine runner

In the following, an analytical approach of the influence of runner rotation on the natural frequencies will be given. Next, the principle of the coupled fluid-structure-interaction simulation is shown. In section 4 a comparison of numerical results of natural frequencies with analytical and experimental ones is performed on two examples. Finally, this topic is concluded.

2. ANALYTICAL MODEL

An analytical model has been developed by Kubota et al. [2] for a uniform annular plate (inner radius r_1, outer radius r_2, thickness h, density ρ_D) with a small ring width ($r_1 \approx r_2$), see Fig. 2. Here, the disc is placed on a fluid-filled cylindrical container.

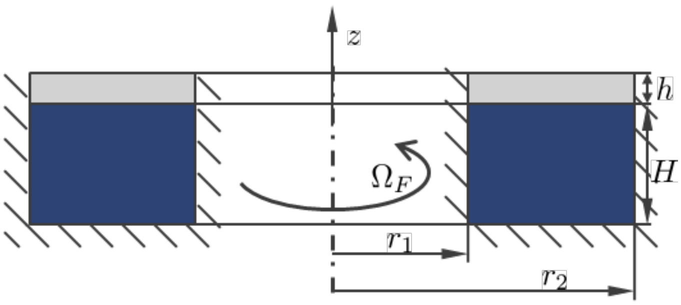

Fig. 2: Sketch of the FSI system

The fluid (density ρ_F) is assumed to be inviscid and is rotating uniformly at rotation speed Ω_F, which is in detail the relative rotation between the fluid and the disc. Therefore, the absolute rotation of the disc is contrary to Ω_F. Further, it is assumed within this model that no motion of the fluid in radial direction is present and no leakage flow through the gap between disc and container appears.

Under these assumptions the Laplace equation for the velocity potential Φ holds:

$$\frac{1}{r_0}\frac{\partial^2\Phi}{\partial\varphi^2}+\frac{\partial^2\Phi}{\partial z^2}=0\,. \tag{1}$$

Herein, $r_0=\sqrt{r_1 r_2}$ denotes the geometric mean of the disc's radius. Further, φ and z are the circumferential and vertical coordinates, respectively.
By introducing the relative added mass

$$M_0=\frac{\rho_F r_0}{n\rho_D h}\coth\left(\frac{nH}{r_0}\right) \tag{2}$$

Kubota et al. [2] end up with the natural frequencies of the disc in water $\omega_{n,fl}$ in implicit form

$$(1+M_0)\omega_{n,fl}^2+2n\Omega_F M_0\omega_{n,fl}+n^2\Omega_F^2 M_0-\omega_{n,air}^2=0 \tag{3}$$

for each diametrical mode $n,\ n=\ldots,\ -3,\ -2,\ -1,\ 1,\ 2,\ 3,\ldots$.
Herein, $\omega_{n,air}$ denotes the natural frequency of mode n in air. If n is negative the mode shape rotates in rotational direction of the fluid (against rotational direction of disc). Otherwise, the rotation of the mode shape is in the same direction as the rotation of the disc.
Based on analytical solution of Kubota et al. [2] Presas et al. [1] extended this solution to a disc that is located between two fluid domains, cf. Fig. 3.

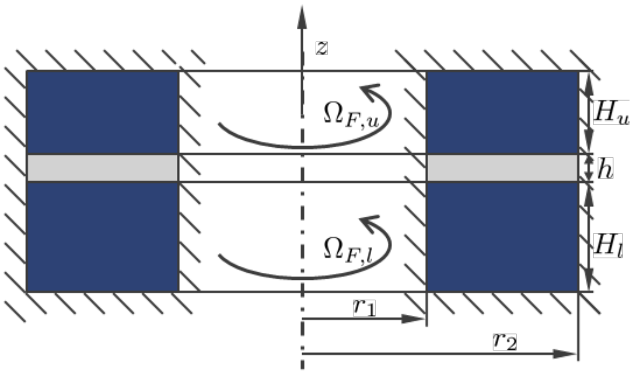

Fig. 3: Sketch of FSI-system with fluid on both sides of the disc

By consideration of the relative added mass of the upper fluid domain

$$M_{0,u}=\frac{\rho_F r_0}{n\rho_D h}\coth\left(\frac{nH_u}{r_0}\right) \tag{4}$$

as well as the lower fluid domain

$$M_{0,l}=\frac{\rho_F r_0}{n\rho_D h}\coth\left(\frac{nH_l}{r_0}\right) \tag{5}$$

the natural frequencies of the disc in water environment are obtained by solution of

$$(1+M_{0,u}+M_{0,l})\omega_{n,fl}^2+2n(\Omega_{F,u}M_{0,u}+\Omega_{F,l}M_{0,l})\omega_{n,fl}+n^2(\Omega_{F,u}^2M_{0,u}+\Omega_{F,l}^2M_{0,l})-\omega_{n,air}^2=0\,. \tag{6}$$

Herein, $\Omega_{F,u}$ denotes the relative rotational velocity of the upper fluid and $\Omega_{F,l}$ is the rotational velocity of the lower fluid.
For both equations (3) and (6) the added mass effect on the natural frequencies for resting fluid ($\Omega_F=0$) is given by (e.g. for Eq. (3)):

$$\omega_{n,fl}^2=\omega_{n,air}^2/(1+M_0)\,. \tag{7}$$

For the relevant mode shapes of hydraulic runners $n^2\Omega_F^2 M_0 << \omega_{n,air}^2$ (cf. Eqs. (3) and (6)) holds. Therewith, the relative rotation of the fluid to the disc causes a term which is linear with respect to $\omega_{n,fl}$. As a consequence two different frequencies are obtained for modes $\pm n$ in rotating fluid environment.

3. FSI-SIMULATION

For the simulation of the rotating disc in water environment the commercial software ANSYS® Mechanical [4] combined with ANSYS® CFX [4] are used. On both, a mechanical model for the disc as well as a computational fluid dynamics model for the fluid domain has to be set up. In both models the coupling interfaces for exchange of displacements and forces have to be defined. Of course, for the remaining surfaces loads and boundary conditions have to be specified.

For the coupled FSI-simulation an incremental iterative procedure is applied. The total simulation time is divided into a certain number of time steps. Within each time step the coupling between fluid and solid domain is performed iteratively. In Fig. 4 the required simulation steps are illustrated.

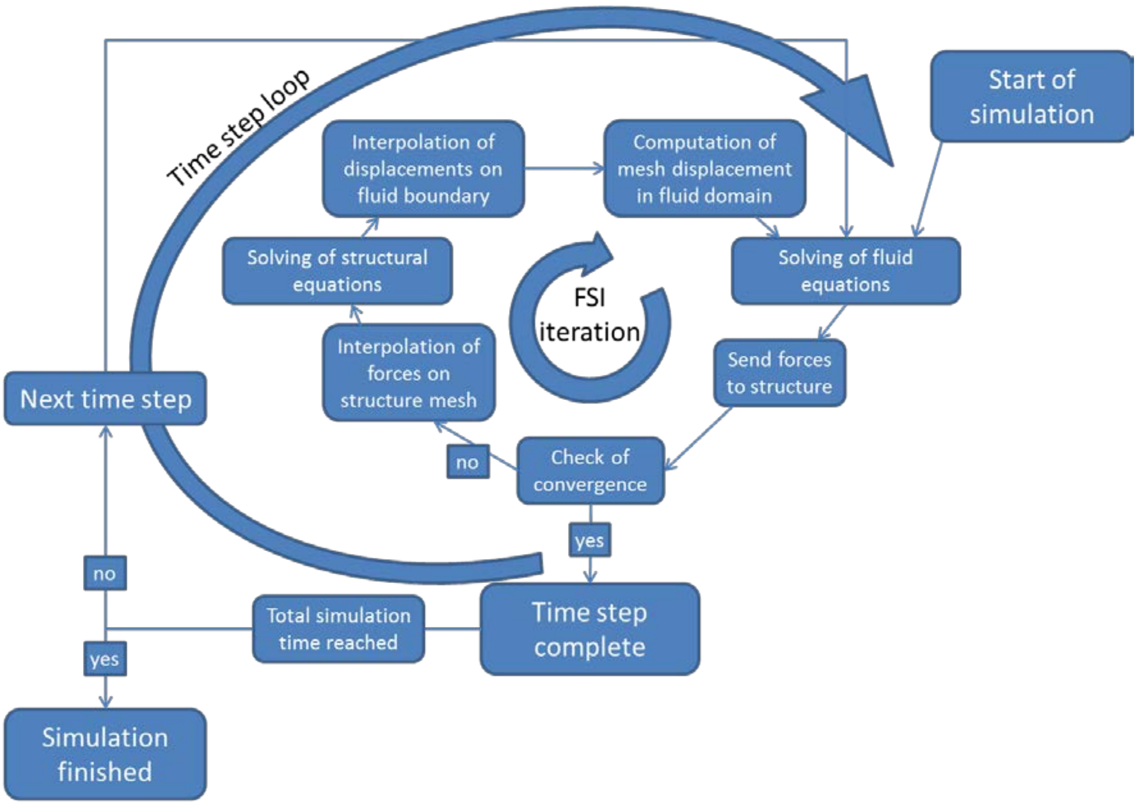

Figure 4: Scheme of coupled fluid-structure interaction simulation

The iterative determination of the fluid structure interaction is started with solution of the structural behavior or with the solution of the fluid flow. In the present context the second option has been chosen. After the determination of pressure distribution and velocity field in the fluid domain, the forces at the interface are sent to the structural domain. In case of convergence of displacements and forces at the interface it is continued with the next time step. Otherwise, the iteration loop is continued. In this case the forces are mapped on the structure and the structural equations of motion are solved. The resulting displacements are

sent back to the fluid domain, where the discretization at the interface is modified and the mesh inside the fluid domain is adjusted for the update of the fluid field.
The simulation ends as soon as the total simulation time is reached.
During the simulation the behavior of the fluid field as well as the deformation can be monitored.

4. EXAMPLES

Two examples are chosen to compare the numerical obtained results with analytical as well as experimental ones. Within the first example a disc on top of a fluid domain is analyzed. For this example analytical results are available. Secondly, the behavior of a test rig is simulated. Here, experimental data is available for verification of numerical results.

a. DISC ON TOP OF FLUID

A disc on top of a fluid container is analyzed, cf. Fig. 2. For the surfaces of the fluid domain FSI-interfaces and no-slip walls are prescribed. The disc is fixed at the inner radius. The remaining surfaces are free or in contact with the fluid, respectively.
The natural frequencies of the disc in air as well as in contact with resting fluid[1] are given in table 1.

n	f_{air}	f_{FSI}
1	167.4 Hz	86.6 Hz
2	271.1 Hz	181.7 Hz
3	600.8 Hz	446.8 Hz
4	1050.1 Hz	825.4 Hz

Table 1: Natural frequencies of resting disc for example 1

For the two-way fluid-structure-coupled simulation the disc is loaded with four forces for 0.01 seconds in order to obtain a n=2 mode shape, cf. Fig. 5. Then, the forces are withdrawn and free vibration of the disc in contact with water is simulated. The simulation is carried out in the rotating frame of reference. Several calculations with different rotating speeds of the disc are performed.

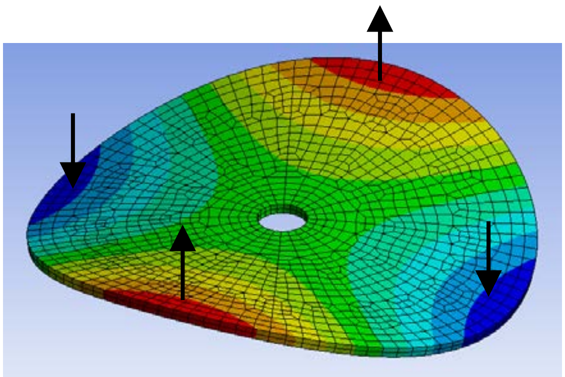

Fig. 5: n=2 mode shape

[1] The natural frequencies are calculated with the Finite Element Method using acoustic Finite Elements for discretization of the fluid domain.

Beside the oscillation of the n=2 mode shape a rotation against the rotating direction of the disc is observed. This motion results from the superposition of the two modes n=±2, which have different frequencies due to the rotation of the fluid.
For evaluation of the simulation the function

$$u(\varphi,t) = A(t)\sin(n\varphi + \varphi_0(t)) \tag{8}$$

is fitted for each time step at the center line of the outer circumferential surface of the disc. Therewith, the amplitude versus time $A(t)$ as well as phase angle versus time $\varphi_0(t)$ curves are obtained. From amplitude behavior the frequency of the oscillation $\omega_{n,0}$ can be derived and from the phase angle the rotational frequency $\Delta\omega_n$ is determined. Therewith, the natural frequencies of forward and backward waves are calculated by

$$\omega_{\pm n,fl} = \omega_{n,0} \mp \Delta\omega_n \,. \tag{9}$$

Table 2 summarizes the results of the performed simulations.

f_{disc}	$\Omega_F / 2\pi$	$\omega_{n,0} / 2\pi$	$\Delta\omega_n / 2\pi$	$\Delta\omega_{n,ana} / 2\pi$	$\omega_{-n,0} / 2\pi$	$\omega_{+n,0} / 2\pi$
0 Hz	0 Hz	181.6 Hz	0 Hz	0 Hz	181.6 Hz	181.6 Hz
5 Hz	3.03 Hz	181.5 Hz	3.7 Hz	3.4 Hz	185.2 Hz	177.8 Hz
10 Hz	5.96 Hz	181.4 Hz	7.2 Hz	6.7 Hz	188.6 Hz	174.2 Hz

Table 2: Summary of results for disc on fluid domain

Within table 2 f_{disc} denotes the rotational frequency of the disc. Ω_F is the relative rotation of the fluid to the disc, which results from CFD part of FSI simulation. The absolute rotational velocity of the fluid is given by f_{disc}-Ω_F. A good agreement of the natural frequency $\omega_{n,0}$ with the results using acoustic finite elements in table 1 is observed. The results of the shift in the natural frequencies due to rotation $\Delta\omega_n$ show slightly higher values than the analytical obtained shift $\Delta\omega_{n,ana}$ using Eq. (3). Finally, the natural frequencies for the backward $\omega_{-n,0}$ as well as the forward wave $\omega_{+n,0}$ are shown in table 2.
By comparing the natural frequencies in water environment, a very good agreement in natural frequencies for mode n=2 between FE approach and FSI simulation is observed, cf. table 1 and table 2. Furthermore, the shift in the natural frequency is slightly higher calculated with the FSI simulation than using the analytical equations.

b. DISC WITHIN TEST RIG

Within the second analysis a real test rig is simulated, which is described in detail in [3]. The model for the FSI simulation is shown in Fig. 6. The disc is fixed at a shaft and is completely surrounded by fluid. The shaft is considered up to the top of the fluid container. At the cutting plane its displacements are fixed. For the fluid domain no-slip walls are prescribed at the surfaces in contact with the container. The surfaces in contact with the disc and the shaft are considered as Fluid-Structure-Interaction surfaces.

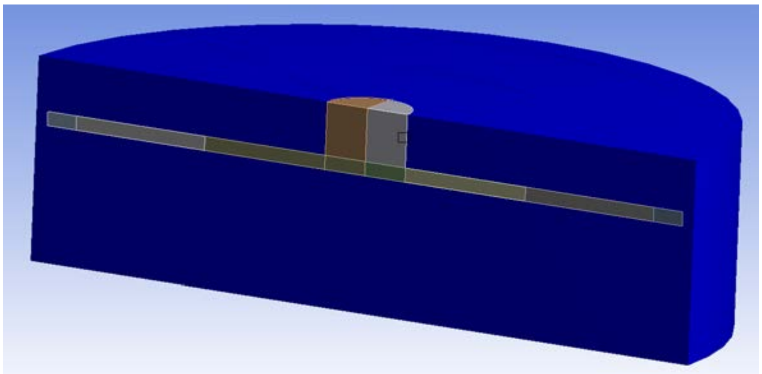

Fig. 6: Sketch of numerical model of test rig

In table 3 the natural frequencies in air and water environment without rotation of the disc are noted. Beside the numerically obtained ones the experimentally determined natural frequencies are noted.

n	f_{air}	f_{FSI}	$f_{air,exp}$ [3]	$f_{FSI,exp}$ [3]
1	164.7 Hz	81.9 Hz	-	-
2	267.0 Hz	159.5 Hz	257.8 Hz	150.5 Hz
3	589.7 Hz	394.0 Hz	588.3 Hz	383.3 Hz
4	1030.4 Hz	739.1 Hz	1031.5 Hz	733.3 Hz

Table 3: Natural frequencies of the resting disc in test rig

Two simulations are performed for the coupled FSI problem. Within the first simulation a n=2 mode is excited in the same way as in the former section. Secondly, a n=3 mode is excited by distributing 6 forces along the circumference. For the n=2 mode a constant time step of 5.e-5 seconds is chosen and for the n=3 mode the time step is specified as 2.5e-5 seconds due to the higher oscillating frequency of this mode. For both simulations a rotational speed of the disc of f_{disc}=8 Hz is considered.
In table 4 the numerically obtained natural frequencies in water environment are compared to the experimental ones.

n	$\omega_{-n,0}/2\pi$	$\omega_{+n,0}/2\pi$	$\omega_{-n,0,\exp}/2\pi$ [3]	$\omega_{+n,0,\exp}/2\pi$ [3]
2	164.0 Hz	152.8 Hz	156.0 Hz	143.4 Hz
3	400.7 Hz	386.1 Hz	390 Hz	374 Hz

Table 4: Natural frequencies of a disc in test rig with rotation

A very good agreement is observed with respect to the spread of the natural frequencies between numerical and experimental results. For the n=2 mode a spread of approximately 12 Hz and for the n=3 mode 15 Hz are obtained. Therefore, the effect of the rotating fluid is captured well by the numerical simulation. The absolute values of the natural frequencies are slightly higher. This behavior is already recognized within the natural frequencies of the resting disc as can be seen from table 3. Thus, this difference in absolute values is related more to the model itself than to the effect of flowing water.

5. CONCLUSION

The influence of flowing water on natural frequencies of a disc-like structure has been investigated within this proceeding. It has been shown that the non-resting fluid results in a split up of diametrical modes in a forward and a backward moving mode with different frequencies. The coupled FSI-simulation is able to capture this effect. Moreover, a very good agreement between numerical results and analytical as well as experimental ones is observed. For a better understanding of this effect and application to hydraulic runners further investigation based on the analytical solution is required since coupled FSI-simulation is very time consuming.

6. REFERENCES

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[4] ANSYS®, Release 14.5.7, ANSYS, Inc., Canonsburg, USA.

7. NOMENCLATURE

H	(m)	height of fluid container
ρ_F	(kg/m³)	density of fluid
Ω_F	(1/s)	relative rotation of fluid
r_1, r_2	(m)	inner, outer radius of fluid container/disc
r_0	(m)	average radius of fluid container/disc
h	(m)	thickness of disc
ρ_D	(kg/m³)	density of disc
Φ	(m²/s)	velocity potential
φ, z	(rad, m)	circumferential, vertical coordinate
M_0	()	relative added mass
n	()	number of diametrical mode
$\omega_{n,air}$, $\omega_{n,fl}$	(1/s)	n^{th} natural frequency in air/fluid environment
$\omega_{n,o}$	(1/s)	n^{th} natural frequency in resting fluid environment
$\Delta\omega_n$	(1/s)	frequency shift of diametrical mode n
f_{disc}	(Hz)	rotational frequency of disc
f_{air}, f_{FSI}	(Hz)	natural frequency of disc in air/fluid environment
u	(m)	displacement field of disc
A	(m)	amplitude of displacement of disc