ON THE HUB-TO-SHROUD RATIO OF AN AXIAL EXPANSION TURBINE FOR ENERGY RECOVERY

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ABSTRACT

The paper presents an example of applying the theory for pipe swirling flow with stagnant region to the axial expansion turbines. It is shown that, for most operating points examined, the hub radius exceeds the stagnant region radius, thus the flow detachment from the hub is avoided. However, for the full-load operating points (large discharge and head), the stagnant region radius is slightly larger than the hub radius. The main strength of the theory assessed in this paper is that it allows reliable computation of the hub radius in the early design stages, before actually designing the blades, without resorting to empirical correlations.

KEYWORDS

Axial expansion turbine, swirling flow, stagnant region

1. INTRODUCTION

In many industrial processes working fluids are needed at high pressure levels. At the end of the processes the fluid is often released to ambient pressure, thus losing the excess of pressure energy. An axial expansion turbine (AXENT) is one possibility to convert the pressure energy (head) into electrical energy. The re-use of recovered energy obviously leads to an overall increase in the system efficiency.

A key requirement for recovering systems is that the energy recovery does not hinder the plant safety or the process stability. The main advantage of the axial expansion turbine is

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the constant flow rate even in the case of power failure, when the turbine reaches its runaway speed. The constant flow rate prevents a hydraulic pressure surge, which could result in severe damage of valves, sealing, and other structures within the system. The AXENT needs no additional safety device or auxiliary control equipment [1]. A particular feature of these axial turbines is the large hub radius with respect to the shroud radius, resulting in short blade span with respect to the blade chord, as it can be seen in Figure 1. The unsteady flow associated with the stator-runner interaction in the AXENT turbine was investigated both experimentally and numerically in [2], and the corresponding numerical methodology was further developed and adapted for parallel computing in [3]. The unsteady forces generated by the stator-rotor interaction are computed and validated in [4].

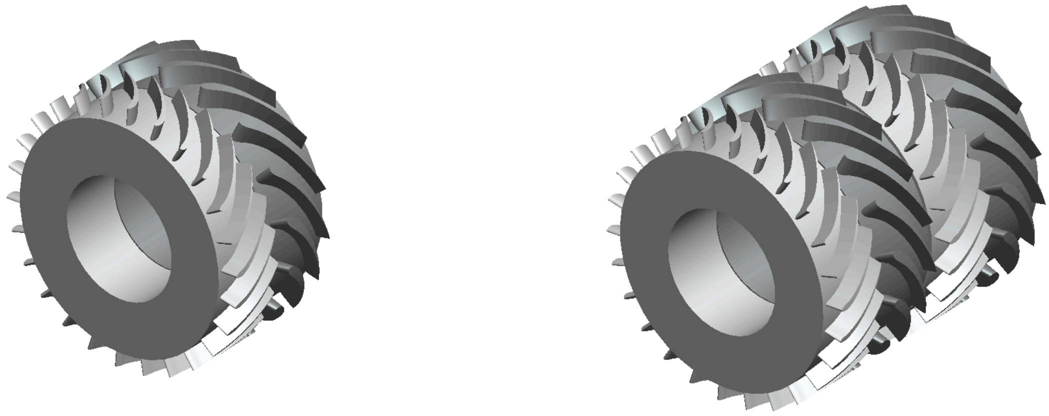

Fig.1 Single- and two-stage configurations of the AXENT turbine.

Although such detailed flow features are significant in the effort of refining the design, we focus in this paper on a basic issue related to the choice of the hub radius, or more precisely the hub-to-shroud radii ratio. The axisymmetric swirling flow computations for decelerated swirling flow in the discharge cone of hydraulic turbines have revealed the necessity of a stagnant region model in addition to the regular turbulent flow solver to improve the agreement with experimental data [5]. As a result, we further developed a model for inviscid swirling flows with stagnant region using a novel variational formulation [6] which evolved in a complete theory for pipe swirling flows [7]. This is the theoretical framework used in this paper to examine the hub radius for the AXENT turbine, in order to validate the theoretical predictions as well as to assess its usefulness for designing the hydraulic turbines. While empirical correlations are usually employed for a first guess of the hub radius for axial turbomachines, eventually followed by a set of successive corrections after designing the blades, the theory for swirling flows with stagnant region provide a rigorous and robust alternative for computing the hub radius before actually designing the blades.

Section 2 summarizes the swirling flow model for application purposes, and Section 3 presents an application of the model for the AXENT turbine. The paper conclusions are summarized in Section 4.

2. SWIRLING FLOW WITH STAGNANT REGION

Susan-Resiga et al. [7] developed a general theory for swirling flow in pipes (the so-called columnar swirling flows, with vanishing radial velocity component) by assuming that a stagnant region could develop within the central region, near the axis. Within the stagnant region all velocity components vanish and at the boundary of such stagnant region the velocity components may have a jump (vortex sheet) while the static pressure remains continuous. The pressure continuity across the vortex sheet that bounds the stagnant region is a requirement for the equilibrium of a fluid interface. The variational formulation corresponding to such swirling flows with stagnant region has a simple and intuitive physical interpretation: *the stagnant region radius corresponds to the minimum of the swirl number*.

Let us revisit the simplest swirling flow that has a constant circulation RV_u and a constant total pressure $P_{\mathrm{tot}} = P + \rho\left(V_{\mathrm{a}}^2 + V_{\mathrm{u}}^2\right)/2$. Note that the radial velocity component is considered negligible. The Euler equations lead to the simple solution for this swirling flow, namely $V_{\mathrm{a}} = \mathrm{constant}$. The "classical" solution for such swirling flow with $V_{\mathrm{u}} \sim 1/R$ says that the circumferential velocity becomes infinite as we approach the axis, with a corresponding infinite negative pressure, and it is the viscosity that takes care of this rather unphysical singularity. However, if we allow a stagnant region to develop near the axis such singularities are no longer occurring.

For the swirling flow in a pipe with radius R_{p}, let us consider the average discharge velocity $V_{\mathrm{ad}} \equiv Q/\left(\pi R_{\mathrm{p}}^2\right)$ when the flow occupies the whole pipe cross-section. The circumferential velocity at the pipe wall, $V_{\mathrm{up}} \equiv \left(RV_{\mathrm{u}}\right)/R_{\mathrm{p}}$ is fixed for a given constant circulation. Note that both V_{ad} and V_{up} are independent of the stagnant region radius, thus we can define the dimensionless swirl intensity parameter,

$$\sigma \equiv \frac{V_{\mathrm{up}}}{V_{\mathrm{ad}}}. \tag{1}$$

If the (unknown) stagnant region is R_{s}, then we denote the stagnation-to-pipe radii ratio squared by,

$$x \equiv \left(\frac{R_{\mathrm{s}}}{R_{\mathrm{p}}}\right)^2. \tag{2}$$

For a swirl with constant circulation and constant total pressure in a pipe we have derived [7] an algebraic (polynomial) equation that relates x and σ as,

$$\frac{\sigma^2}{x^2} - \frac{2}{\left(1-x\right)^3} = 0. \tag{3}$$

For non-vanishing σ the solution of Eq. (3) provides the stagnant region extent. When $\sigma = 0$ there is no rotation and the flow occupies the whole pipe cross-section. Figure 2 shows the variation of $\sqrt{x}$ versus σ, revealing a sharp increase in the stagnant region radius for small swirl intensity followed by a slower increase as σ gets large.

Let us now suppose that the pipe has a central cylindrical body. We have proved in [7] that if the radius of the central body is smaller than the value indicated in Fig. 2, for the corresponding swirl intensity, then the radius of the stagnant region remains unchanged. On the other hand, if the radius of the central body is larger than the stagnant region there will be no stagnant region and the flow occupies the whole annular section from the central body to the pipe wall. These results are useful when applying the above theory to the determination or validation of the hub radius of an axial turbomachine.

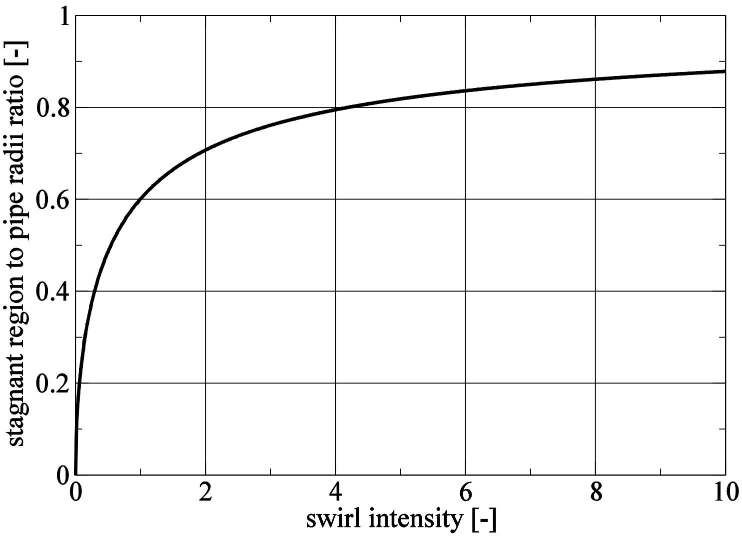

Fig.2 The ratio between the stagnant region radius and the pipe radius versus the swirl intensity, for a swirl with constant circulation and constant total pressure.

3. THE HUB RADIUS FOR THE AXENT TURBINE

Let us examine now an axial expansion turbine (AXENT) with shroud (pipe) radius $R_{\text{shr}} = 110\,mm$ and a hub radius $R_{\text{hub}} = 95\,mm$. Numerical analysis of the flow in the bladed regions provide the turbine head H versus the discharge Q and runner speed n as shown in the first three columns of Tab. 1 for a single stage turbine, and Tab. 2 for a two-stage turbine, respectively.

In order to apply the theory from §2 we have to compute the swirl intensity parameter σ defined in Eq. (1). The fundamental equation of turbo-machines (Euler equation) gives,

$$gH = UV_{\text{u}}, \text{ or } gH = \frac{\pi n}{30} R_{\text{p}} V_{\text{up}} \Rightarrow V_{\text{up}} = \frac{30}{\pi}\frac{gH}{nR_{\text{p}}}. \tag{4}$$

The average discharge velocity is

$$V_{\text{ad}} = \frac{Q}{\pi R_{\text{p}}^2}. \tag{5}$$

In both Eqs. (4) and (5) the pipe radius is practically the shroud radius for the turbine, i.e. $R_{\text{p}} = R_{\text{shr}}$. As a result, the dimensionless swirl intensity parameter is:

$$\sigma \equiv \frac{V_{\text{up}}}{V_{\text{ad}}} = 30\frac{gHR_{\text{shr}}}{nQ}. \tag{6}$$

Table 1 shows in the fourth column the σ values computed for each operating point. For the two-stage turbine, the σ values shown in Table 2 are computed by halving the overall head, thus assuming that the head is evenly distributed for each turbine stage.

Solving the Eq. (3) we obtain the stagnant region radius,

$$R_{\text{s}} = \sqrt{x}R_{\text{shr}}, \tag{7}$$

as shown in the last column of both Tab. 1 and Tab. 2

$n\left[\mathrm{rpm}\right]$	$Q\left[\mathrm{m}^3/\mathrm{s}\right]$	$H\left[\mathrm{m}\right]$	$\sigma\left[-\right]$	$R_s\left[\mathrm{mm}\right]$	
1500	0.050	10.2	4.403	88.6	
1500	0.060	16.8	6.043	92.1	
1500	0.065	20.3	6.740	93.2	
1500	0.070	24.0	7.400	**94.0**	Design point
1500	0.075	27.9	8.028	94.8	
1500	0.080	32.0	8.633	95.4	
1500	0.090	40.8	9.784	96.5	
3000	0.100	40.4	4.359	88.5	
3000	0.110	53.0	5.199	90.5	
3000	0.120	66.3	5.962	91.9	
3000	0.125	73.2	6.319	92.5	
3000	0.130	80.3	6.665	93.0	
3000	0.135	87.6	7.002	**93.5**	Design point
3000	0.140	95.0	7.322	93.9	
3000	0.145	102.6	7.635	94.3	

Tab. 1 Single stage axial expansion turbine data.

$n\left[\mathrm{rpm}\right]$	$Q\left[\mathrm{m}^3/\mathrm{s}\right]$	$H\left[\mathrm{m}\right]$	$\sigma\left[-\right]$	$R_s\left[\mathrm{mm}\right]$	
1500	0.050	19.4	4.187	88.0	
1500	0.060	33.2	5.971	91.9	
1500	0.070	48.0	7.400	**94.0**	Design point
1500	0.075	55.9	8.043	94.8	
1500	0.080	64.0	8.633	95.4	
1500	0.085	72.4	9.191	95.9	
1500	0.090	81.2	9.736	96.4	
3000	0.100	76.5	4.127	87.8	
3000	0.110	103.3	5.067	90.2	
3000	0.120	131.2	5.899	91.8	
3000	0.130	160.1	6.645	93.0	
3000	0.135	174.9	6.990	**93.5**	Design point
3000	0.140	190.0	7.322	93.9	
3000	0.145	205.4	7.643	94.3	

Tab. 2 Two-stage axial expansion turbine data.

Figure 3 shows a synoptic comparison between the stagnant region radius (divided by the shroud radius) and the dimensionless hub radius $R_{\mathrm{hub}}/R_{\mathrm{shr}} = 0.864$. One can see that for most of the operating points the hub radius is larger than the stagnant region radius, thus the stagnant region does not appear. However, there are several operating points for large discharge and head values where the stagnant region radius is slightly larger than the hub radius. This is quite acceptable since the theory considers an inviscid flow, and the above difference might be very well of the same order as the boundary layer displacement thickness.

Remarkably, the stagnant region radius computed for the design operating points is just marginally smaller than the actual hub radius, proving that the theory from §2 can be reliably used for design purposes.

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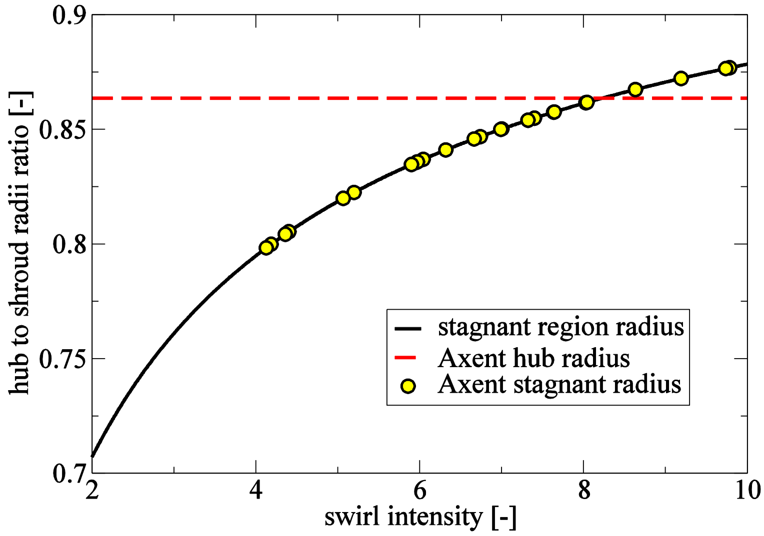

Fig.3 The ratio between the stagnant region radius and the pipe radius versus the swirl intensity, for the axial expansion turbine AXENT, both single stage and two stages.

4. CONCLUSIONS

The paper presents an analysis of the swirling flow in an axial expansion turbine (AXENT) from the perspective of the radial extent of the stagnant region with respect to the hub radius. It is shown that the stagnant region radius predicted by the theory of swirling flow in pipes is quite consistent with the hub radius of the AXENT turbine, and the stagnant region development is actually avoided for most of the examined operating points. It is only at the operating points with large discharge and head values (full load) where a stagnant region could marginally develop near the hub.

However, the main strength of the theory for pipe swirling flows with stagnant region is that one can correctly choose the hub radius in the early design stages, before actually designing the blades. As a result, there is no need to use empirical formulae or trial and error approaches, as one can accurately compute the required hub radius to avoid flow detachment from the hub, and check the result within a wide range of operating points.

5. ACKNOWLEDGEMENTS

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7. NOMENCLATURE

R	[m]	radius	V_{ad}	[m/s]	average discharge velocity
R_p	[m]	pipe radius	V_{up}	[m/s]	tangential velocity at pipe wall
R_s	[m]	stagnant region radius	σ	[-]	swirl intensity
R_{hub}	[m]	hub radius	x	[-]	inner/outer radii ratio
R_{shr}	[m]	shroud radius	n	[rpm]	runner speed
V_a	[m/s]	axial velocity	Q	[m^3/s]	volumetric flowrate
V_u	[m/s]	tangential velocity	H	[m]	turbine / stage head
P	[Pa]	static pressure	g	[m/s^2]	gravity